The University of Queensland

School of Engineering

DESIGN ENGINEERING NOTES

GEOMETRIC OR KINEMATIC DESIGN

Mechanical failure is generally interpreted in terms of change, some physical or chemical process having produced failure. However the improper location and guidance of components also constitutes an important mode of failure often ignored in mechanical engineering practice. Few machine design texts even mention the subject. These notes* describe the theoretical concepts underlying good practice in geometric or kinematic design and give some examples of its application.

THEORETICAL CONCEPTS

Degrees of freedom

A rigid body whose position is not restrained in any way has 6 ‘degrees of freedom’. Defining motion as composed of translation and rotation, a body perfectly free in space has 3 independent translations parallel to 3 mutually perpendicular (Cartesian) axes, and 3 independent rotations about those axes. Put another way, 6 numbers are necessary to define completely the position of a body in space: 3 rectangular coordinates and 3 angles.

Constraints

One obstruction, or ‘constraint’, is sufficient to prevent any one such motion. One constraint will constrain 1 degree of freedom so that an object must have the same number of constraints as the number of degrees of freedom we wish to prevent. Six maintained contacts are in general sufficient to destroy all 6 degrees of freedom, that is, to completely locate or constrain an object.

Closure

A ‘closure’ is the means by which a constraint is maintained. To distinguish between a constraint and a closure consider a three-legged stool resting on a more or less horizontal surface. The stool will be quite stable and the contacts (3 legs) will have eliminated 3 degrees of freedom, translation along an axis normal to the surface and rotation about 2 perpendicular axes lying in the surface. The same stool in

* Notes originally developed by Assoc Prof J E (Tim) Holt

contact with a vertical surface will no longer be stable and yet 3 contacts remain. The difference between the two situations is of course the force of gravity which for the horizontal case ‘maintains’ the contacts. Such action is termed a ‘force closure’. Alternatively, further contacts may be added to maintain the necessary geometric ones, providing a ‘body closure’. (Failure to distinguish between constraints and closures leads to some sources referring to 12 degrees of freedom.)

Redundancy

While 6 contacts are required to constrain 6 degrees of freedom, 6 contacts are not necessarily sufficient to eliminate 6 degrees of freedom. For example, 3 contacts plus gravity closure are sufficient to enable a stool to sit firmly on a horizontal surface. The addition of a fourth contact, a fourth leg, does not eliminate any of the remaining degrees of freedom, translation along 2 axes in the surface and rotation about an axis normal to the surface. In a geometric sense the fourth leg is ‘redundant’. A consequence of this redundancy is that the four-legged stool will rock when placed on a hard floor. Alternatively, the 4 legs may be forced into simultaneous contact by increasing the closure force, thereby deforming the stool structure.

Conditioning

A further qualification about contacts and constraints is necessary. A contact must be properly conditioned. When a contact surface is normal to the direction of motion the contact is most effective in preventing the motion: the contact is said to be ‘well-conditioned’. On the other hand, surface contacts oblique to the direction of motion are ‘badly conditioned’. For instance, in an extreme case there is some resistance to sliding a stool across a solid floor but the contacts are parallel to that motion and are thus not conditioned to prevent it. The resistance is due only to friction.

THE KELVIN COUPLING

If we wish to completely restrain an object it requires 6 constraints, one for each of the 6 degrees of freedom, and at least one closure. A classical example of such a system is the hole, slot and plane method pictured in Figure 1, referred to as the Kelvin Coupling. It is a classical method in the sense that while it is not generally known or applied in this form, the principles that it embodies will be followed in all everyday good mechanical design.

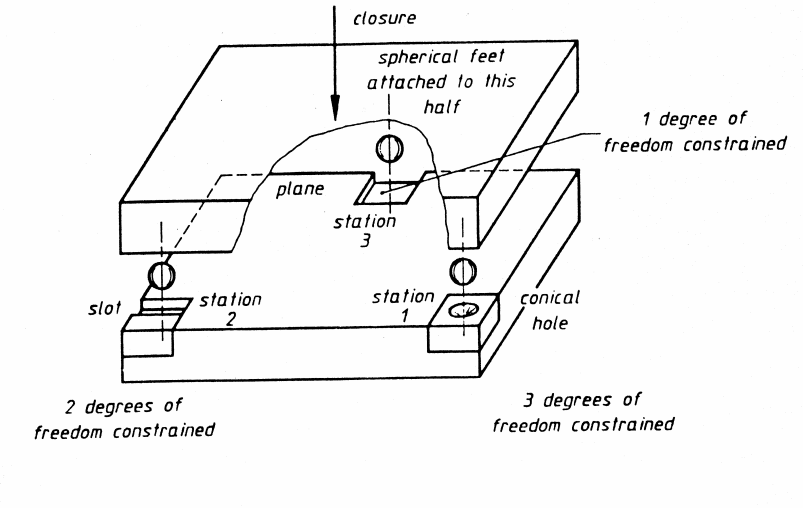


Figure 1. The Kelvin Coupling

At station 1, a sphere on the upper half fits into a conical hole in the lower. There would be no geometrical change in the contact conditions if the conical hole was replaced by a tetrahedral hole providing three flat sloping surfaces for location. Three degrees of freedom of the upper half of the coupling have been eliminated at this station. Although the upper half remains free to rotate about any axis, it has no translational freedom.

Consider now the engagement of the second sphere in a slot or Vee-groove at station 2. There are 2 points of contact, one on each side of the slot. The 2 contact points eliminate another 2 degrees of freedom of the upper half, rotation about an axis normal to the lower surface and rotation about an axis in that surface. Only one degree of freedom is left, rotation about the axis of the slot, also passing through the centre of the hole.

At station 3, the third sphere provides one point contact, eliminating the final rotational degree of freedom.

If a single force closure is added, normal to the lower surface as indicated, the location system is now complete. There are no redundancies: 6 points of contact have eliminated 6 degrees of freedom.

GEOMETRIC DESIGN

The principles of geometric design, or kinematic design, derive from this elementary theory. Geometric design aims at providing exactly the right number of independent, well-conditioned constraints maintained by appropriate closure forces to ensure the desired degrees of freedom between 2 rigid components.

Geometric design principles are basic to good design across the whole spectrum of machines. In most machinery, where the loads are certainly not light, the location of one part relative to another will always introduce some redundancy. Kinematic or geometric design aims at a statically determinate system wherein parts are fixed by conditions of equilibrium alone.

METHODS OF LOCATION

In general mechanical engineering applications the 2 most important ways of locating or guiding components relative to each other which satisfy geometric design principles are the (i) 3, 2, 1 and the (ii) 4, 1, 1 systems.

The 3, 2, 1 system

This method begins with the location of one component relative to another on a plane, as shown in Figure 2.

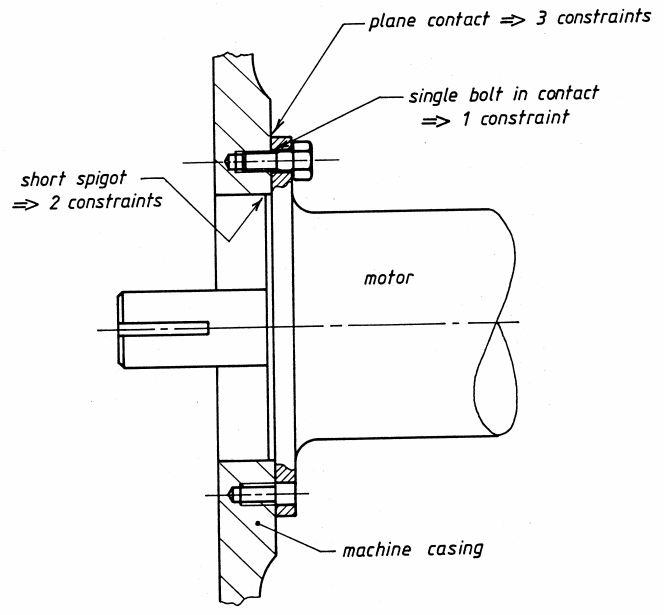


Figure 2. Example of the 3, 2, 1 system: flange mounting.

The primary location is the contact between 2 surfaces. This plane location eliminates 3 degrees of freedom: one translation along the axis of the motor and 2 rotations about 2 perpendicular axes in the plane. Three degrees of freedom remain. The spigot confines 2 more degrees of freedom: 2 translations along 2 perpendicular axes in the plane. One degree of freedom remains, the rotation about its axis. This is constrained by any one of the clamping bolts contacting the side of the bolt hole.

The 3, 2, 1 nomenclature thus describes the number of degrees of freedom constrained by each separate element of the location system. The clamping bolts provide the necessary horizontal closure. Clearance holes are essential for the clamping bolts; of the customary 4 or 6 bolts used in such joints, only one contact is required to suppress the remaining rotational degree of freedom.

Attempts to provide contact on more than one bolt introduces redundancy, leading to a requirement for a perfect fit.

The 4, 1, 1 system

The second method begins with the location of one component relative to another in a cylinder. Figure 3 shows two very common examples.

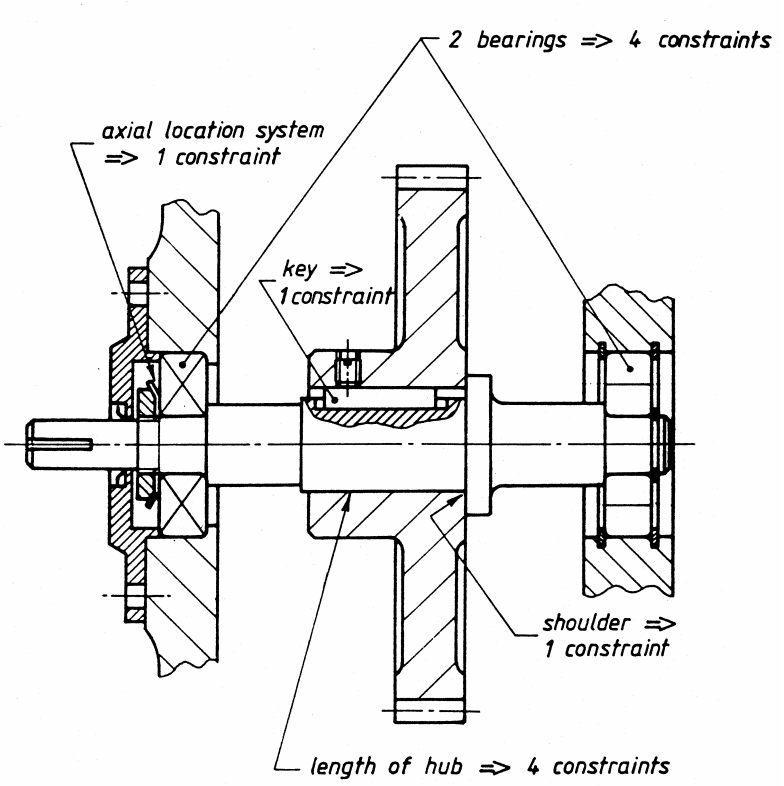


Figure 3. Examples of the 4, 1, 1 system of constraint: hub on a shaft and shaft support.

In each case the primary location is 2 sets of plane contacts on a cylinder. Each plane contact (at each bearing for example) is equivalent to 2 points plus a closure, which constrains 2 translational degrees of freedom.

Two sets of such contacts thus constrain 4 degrees of freedom, 2 of translation and 2 of rotation, leaving one translation along the axis of the shaft and one rotation about that axis. In the shaft-bearing example the second stage of the system prevents shaft translation, usually by fixing one bearing to the shaft and in turn fixing that bearing in the housing. This leaves the shaft free to rotate as usually required. The 3 separate elements of the location system thus make a 4, 1, 1 pattern.

The location of the gear on the shaft is another example of the 4, 1, 1 system. A hub length of about 1.5*d* to 2*d*, where *d* is the diameter of the shaft, establishes the equivalent of 2 spaced bearings (4 constraints), the shaft shoulder and the set screw locate the gear axially and the key locates it against rotation.

The constraint system is virtually independent of the fit between gear hub and shaft. An interference fit on a short hub does not substitute for a suitable length of engagement.

CONSTRAINTS AND LINKAGES

The functional requirements in the examples given in Figures 2 and 3 concern the unambiguous relative location of 2 components. The 3, 2, 1 and 4, 1, 1 systems are equally important in linkages where the need for defined motion is added to location. Figure 4 shows a simple mechanism for engaging and disengaging a jaw clutch between 2 shafts, A and B.

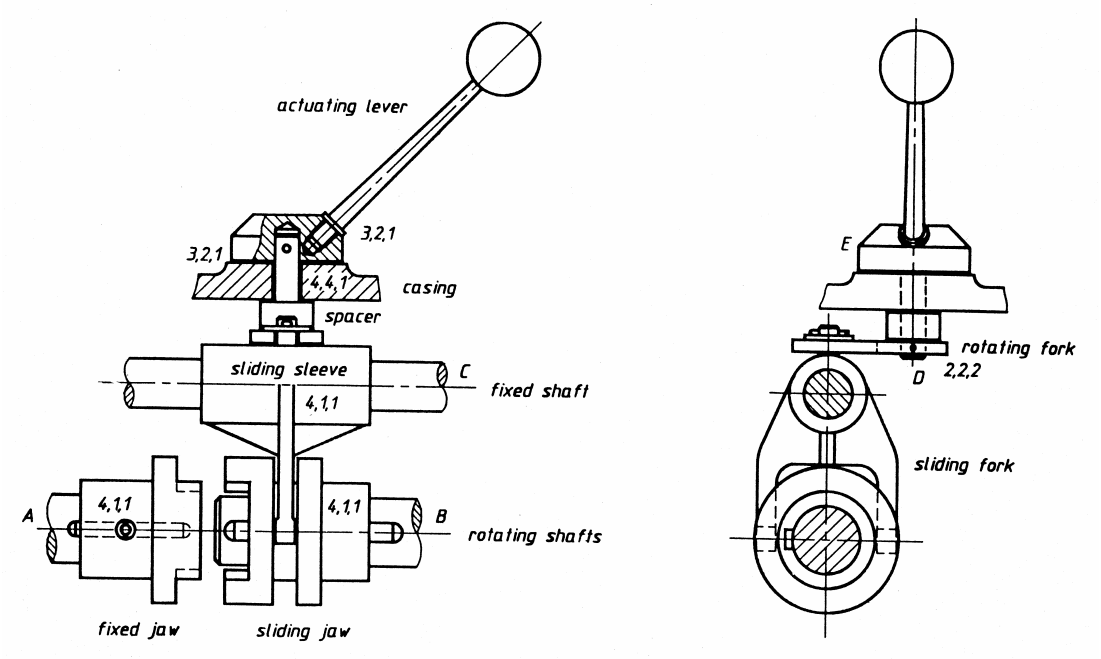


Figure 4, A geometric analysis of a clutch-engaging mechanism.

The rotary motion of the actuating lever is converted into a linear motion to slide one half of the clutch along shaft B. Torque is transmitted between this sliding half and its supporting shaft by means of a long key. The geometric requirements of the linkage may be analyzed as follows:

1) The fixed jaw is the equivalent of the gear hub in Figure 3. The 4, 1, 1 system applies.

2) The sliding jaw has similar requirements except that the contact pads on the sliding fork locate it axially as well as serving to transmit the actuating force. Again a 4, 1, 1 system is needed.

3) The sleeve sliding on the fixed shaft C is directly subjected to overturning moments, due to the force exerted by the rotating fork above to overcome the resistance to sliding of the jaw below. The rotation of the sleeve about the axis of shaft C is eliminated by the fork contacting the bottom of the groove on the sliding jaw, a case of body closure.

4) The contact of the rotating fork with shaft D is equivalent to a single bearing. Two translational degrees of freedom are destroyed. The addition of a pin in the orientation shown constrains the third translation and rotation about the shaft axis. The 2 remaining rotations are eliminated by the contact between the jaw and the sliding sleeve. A

washer and split pin provides a body closure. This component serves to introduce a different, less common location system, the 2, 2, 2 method.

5) The rotating shaft D utilizes the 4, 1, 1 system again, the rotational degree of freedom remaining.

6) The block E is located with a 3, 2, 1 system.

7) The actuating lever is seated on a flat surface (3 constraints) with a short length engaged in a cylindrical hole (2 constraints) and the final rotation prevented by the thread helix. Friction forces provide a closure.

Careful attention to the principles of geometric design for this linkage is not only necessary to ensure its proper functioning. It is also necessary to allow the widest possible tolerances on individual parts and to minimize the effect of the inevitable wear on its operation.

CONCLUSIONS

1. The requirement for adequate location and/or guidance of one component relative to another is ubiquitous in mechanical design. It is fundamental to the proper functioning of machinery.

2. Improper functioning due to inadequate or redundant systems of constraint is just as much a failure mode as the more generally accepted ones.

3. The theoretical basis of geometric design is straightforward; the careful application of that theory should be one of the aims of all competent machine designers.