6.4. Displacement and equivalent displacement volumes

As even engineers have, from time to time, expressed widely differing ideas about the volumetric displacement of the Wankel RC engine, it seems opportune to examine the facts and circumstances to see whether a rational answer is possible. Before entering upon this analysis it ought to be reiterated that the displacement volume of any positive displacement engine has no direct bearing upon power output.

Let p = m.e.p. (lb/in^2)
D = bore diameter (in)
A = effective piston area (in^2)

Force = $F = p \times A$ lb
= pressure × area

L = stroke (ft)
S = total distance travelled by piston in unit time (ft/min)
= mean piston speed (ft/min)
N = crankshaft speed (rev/min)
n = number of cylinders
f = number of working (expansion) phases per crankshaft revolution.
= $\frac{1}{2}$ for four-stroke engine
η = mechanical efficiency (%)

$$\text{Hence b.h.p.} = \frac{F \times S \times \text{Constant} \times \eta}{33{,}000}$$

$$= \left(p \times \frac{\pi D^2}{4}\right)(2LN)\left(\frac{nf}{2 \times 33{,}000}\right)\eta$$

$$= \frac{pD^2S\eta n}{168{,}069} \tag{6.19}$$

where $168{,}069 = \dfrac{16 \times 33{,}000}{\pi}$

According to Equation 6.19, the power output of any positive displacement engine is *the product of the applied force* ($p \times A$) *multiplied by the distance* (S) *travelled by the piston in unit time*. It may be convenient for various purposes, such as for rapid comparison of different engines, to rearrange Equation 6.19 so that it appears to contain the displacement volume, but this expedient cannot overrule the basic facts.

It seems most unwise and unsatisfactory to dismiss the controversy in this summary fashion, as engine size is of considerable interest to the designer who, after all, deals with dimensions and proportions. An analytical approach may, therefore, yield other convincing facts.

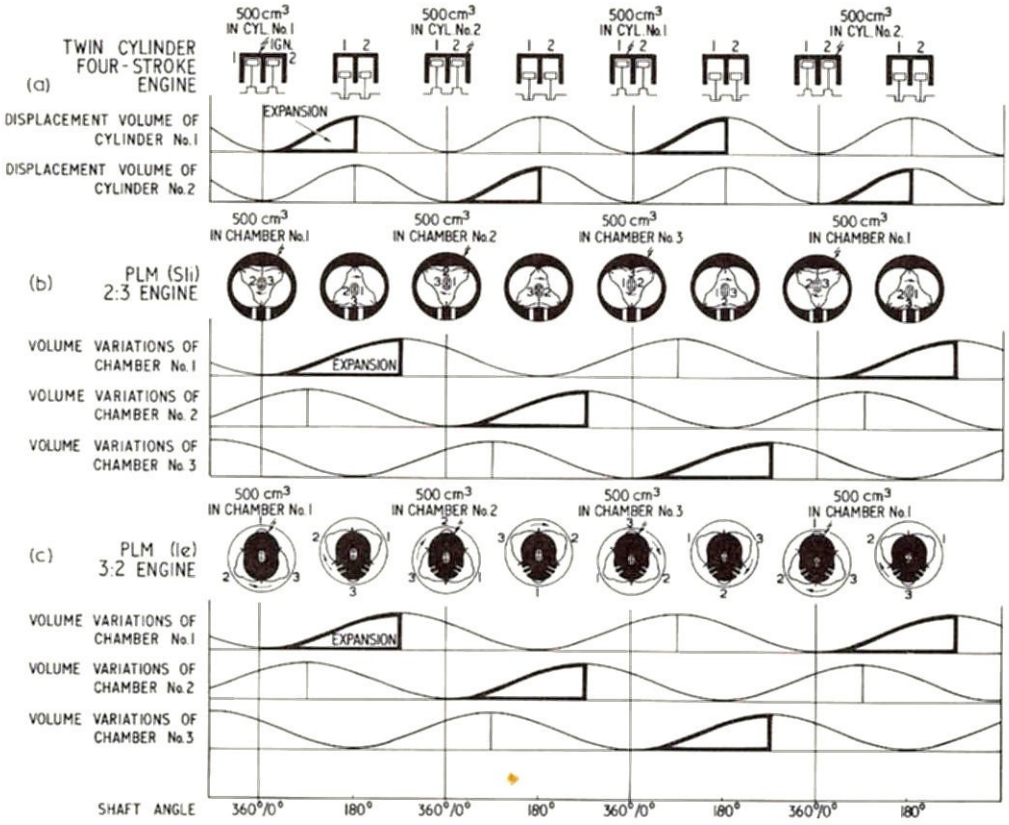


Fig. 6.8 to 6.13. Comparison of the displacement volumes of various Otto-cycle reciprocating piston engines and RC engines relative to crankshaft or eccentric shaft rotation. Parts shown solidly black are stationary, those shown in outline only are the moving pistons or rotors

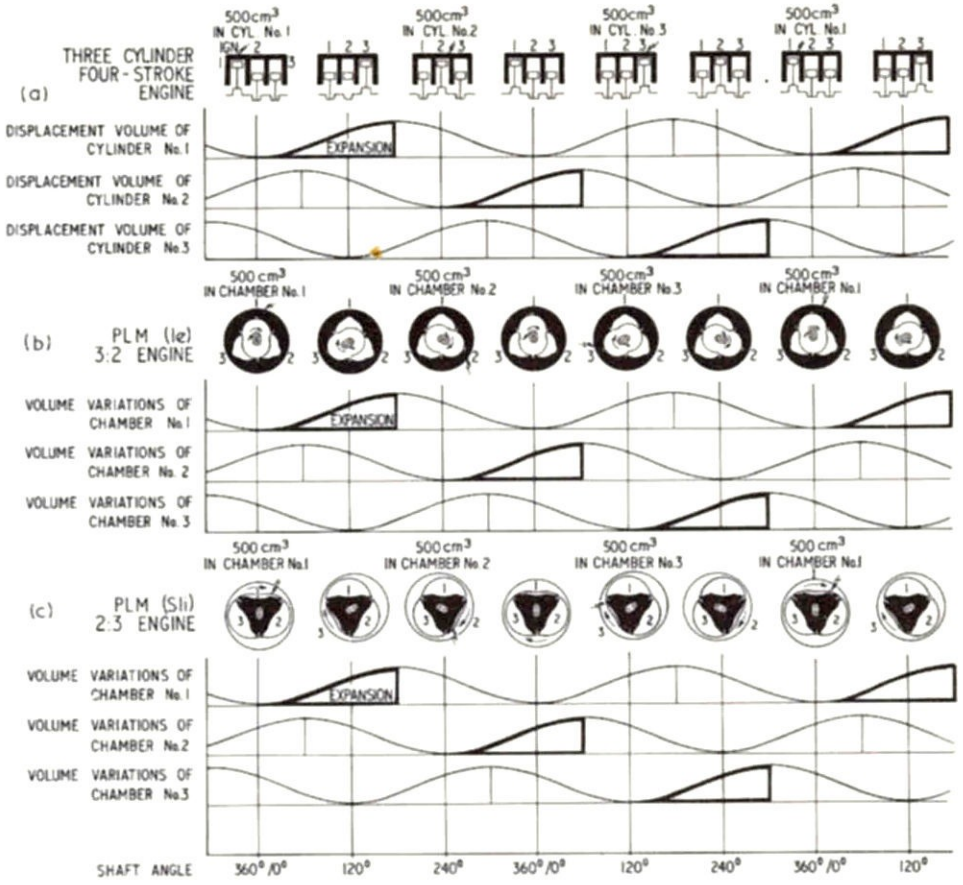


Fig. 6.9.

Engine performance is usually expressed as b.h.p. developed at crankshaft speed, the crankshaft being the first component capable of transmitting the forces exerted by one or more pistons to any other machine. Accordingly this may be considered as the definition of the *primary engine component*.

Similarly the eccentric shaft of a Wankel RC engine is also a primary component as it is the first component capable of transmitting the forces acting on one or more rotors to any other machine. It is, therefore, pertinent to plot or compare the volume displacement curves, which are sinusoidal for reciprocating and rotary piston engines, relative to crankshaft and eccentric shaft rotation respectively.

It is not disputed that a single cylinder two-stroke engine displaces nominally one stroke volume per crankshaft revolution so that it is, in fact, the equivalent of a four-stroke engine displacing twice as much, but the displacement per revolution is generally accepted. Various interpretations could be used to explain this anomaly, the most magnanimous explanation probably being that even scientists and engineers are not always consistent. In any case, the two-stroke engine as it is known today is most unlikely to

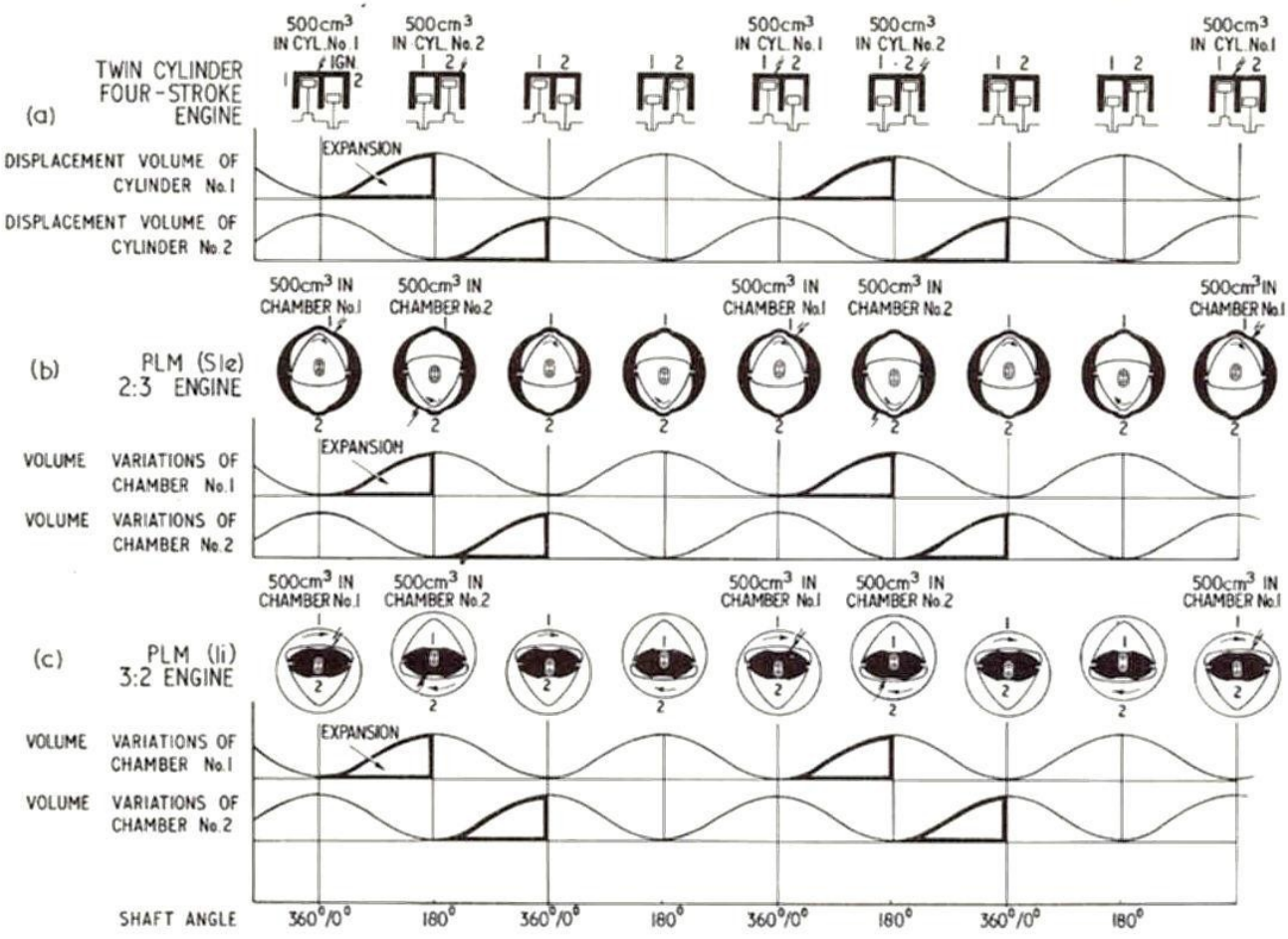


Fig. 6.10.

supersede the four-cycle engine in any major automotive application. It is therefore pointless to revive this particular controversy.

Fig. 6.8a shows the displacement curves of a parallel twin cylinder four-stroke engine, every phase extending nominally over 180° of crankshaft rotation and the actual working or expansion phases being outlined more heavily.

Fig. 6.8b shows the volume variation curves of a single rotor PLM (Sli) 2:3 engine, that is of a Wankel RC type engine. Clearly there are three curves as there are three variable volume chambers. The most notable feature is that every individual phase extends over 270° of shaft rotation – the shaft rotating at three times the speed of the rotor – so that no more than eight complete thermodynamic phases or two complete cycles occur during any two shaft revolutions. Consequently a single rotor Wankel RC engine is indeed the equivalent of a four-stroke twin cylinder reciprocating piston engine whose stroke volume of every piston equals the displacement volume of any flank of the Wankel RC engine.

Fig. 6.8c shows the volume variation curves of a so-called PLM(Ie)3:2 RC engine in which the inner rotor is stationary whilst the outer enveloping rotor moves in the manner of a planetary pinion by rotating about its own centre of gravity and simultaneously orbiting round the output shaft centre. As the sealing elements and curve generating points are accommodated in the outer and power transmitting component, it is the inner

rotor which has the epitrochoidal shape. This engine also has three variable volume chambers and only two complete thermodynamic cycles occur during any two shaft revolutions.

Inverting the design of Fig. 6.8b and c by arranging that the rotors of the configurations shown in Fig. 6.8 are stationary and that the components which were stationary become the working rotors, we obtain the PLM(Ie)3:2 Fig. 6.9b and PLM(Sli)2:3 Fig. 6.9c. Assuming that the respective proportions have remained unaltered, it is evident from the volume variation curves that these two rotary piston engines are without doubt the equivalent of the three cylinder four-stroke engine shown in Fig. 6.9a.

It is emphasised that in proportion and superficially Fig. 6.8b would appear to be the same as Fig. 6.9c and Fig. 6.8c the same as Fig. 6.9b. All four engines have, in fact, three variable volume chambers.

Evidently this comparison underlines the importance of a careful analytical approach to RC engines, as mere counting of chambers will certainly fail to yield the correct answer as underlined by Figs. 6.10–6.13.

Fig. 6.10 and Fig. 6.11 depict RC engines which have ostensibly only two variable volume chambers, yet (as illustrated by the volume variation curves) these engines can also be the equivalent of either twin or three cylinder engines. Even so it is most unlikely

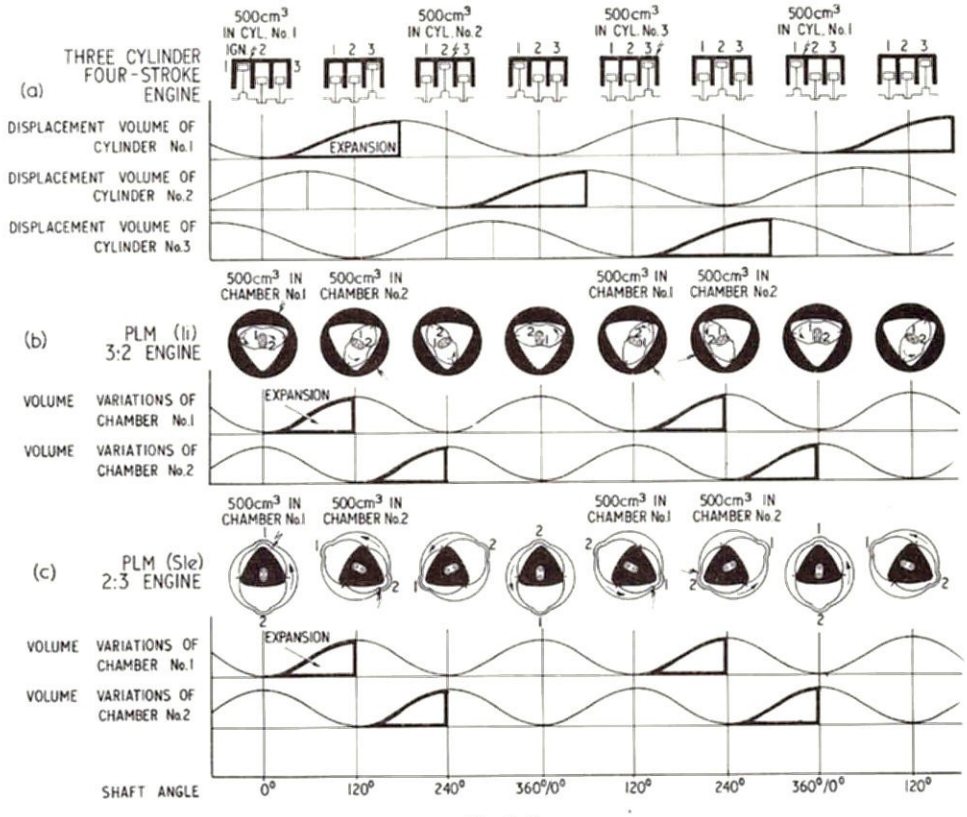


Fig. 6.11.

that any of the engines shown in Fig. 6.10 or in Fig. 6.11b and c will be produced without compelling reasons unconnected with engine design or performance. Consideration of torque and vibration characteristics suggests a different timing of the respective cycles; unfortunately it is impossible to arrange that the RC engines illustrated in Figs. 6.11 and 6.13 have their expansion phases equally spaced. However, they complete six thermodynamic cycles in four shaft revolutions which permits the conclusion that they are, for practical purposes, the equivalent of the three cylinder engines shown.

The RC engines shown in Fig. 6.12 and Fig. 6.13 are so-called single rotation engines,

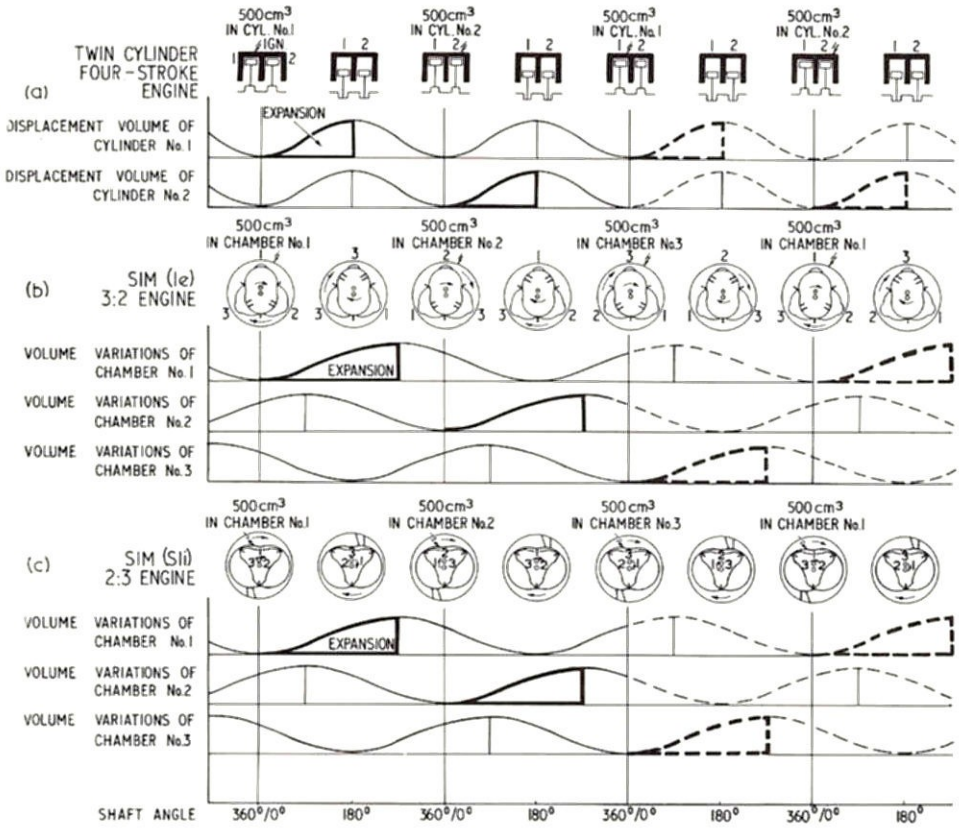


Fig. 6.12.

which means, according to Felix Wankel's book 'Rotary Piston Machines', that the two chambers forming components of the respective configurations rotate about their own centres of gravity.

Ing. D. Eiermann and Ing. H. Kühner, who work in close association with Felix Wankel, have evolved the general formulae for calculating rapidly the true equivalent maximum volume of RC engines possessing any convenient number of working chambers round a single rotor. They point out that non-intersecting trochoids and their complementary

rotors form variable volume chambers. It is immaterial whether these engines are classified as single (SIM) or planetary (PLM) rotation configurations, the respective maximum and minimum volumes will be identical provided ϕ, e, R and B_c of these engines are the same for every design. Moreover, these volumes will be the same for hypotrochoidal and epitrochoidal designs irrespective of their mode of engagement or the position of their trochoid generating points, that is irrespective of whether they are to be denoted ($\overline{\text{SIi}}$), ($\overline{\text{SIe}}$), ($\overline{\text{Ii}}$) or ($\overline{\text{Ie}}$) machines. (For precise definitions of these notations see 'Rotary Piston Machines').

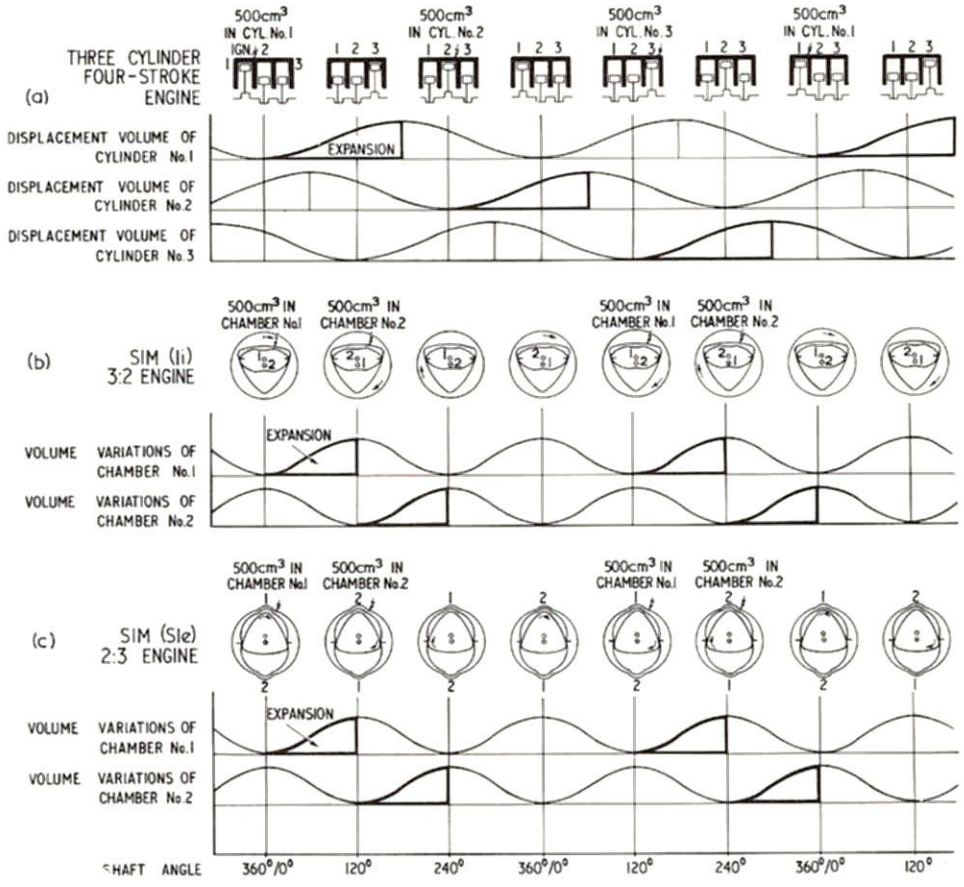


Fig. 6.13.

One method of finding maximum chamber volumes (V_{max}) has already been evolved, nevertheless it may be pertinent to re-examine the conditions and see whether a different approach yields the same results. Stroke × bore cross-sectional area signifies the displacement volume of reciprocating piston engines, and the maximum cylinder volume (V_{max}) is the stroke volume plus the volume contained by the piston and the cylinder head when the piston is in the t.d.c. position. The position is very similar in RC engines except that it is not always so obvious which are the relevant areas and which is the distance to

be taken into account. With reference to Fig. 6.7, the relevant area is represented by any side of the equilateral triangle abc which connects the apexes of the rotor, this area is

$$2(R_1 \sin \pi/3)B_c$$

for the familiar Wankel configuration, or in general terms it is

$$2\left(R_1 \sin \frac{\pi}{Z}\right)B_c \tag{6.20}$$

and the greatest chamber volume

$$V = (A_s + A_{\min} - A_r)B_c = 2\left(R_1 \sin \frac{\pi}{Z}\right)B_c \times (\text{Distance moved by this area})$$

The relevant distance is found as follows:
Assuming a rotor with Z flanks and a stationary trochoid. The eccentric shaft of this configuration must make Z complete revolutions for every single revolution of the rotor. One rotor flank covers $2(Z-1)$ working distances during every rotor revolution. When, therefore, the eccentric shaft turns through an angle α during one working phase

$$\alpha^c = \frac{Z \times 2\pi}{2(Z-1)} = \frac{Z \times \pi}{Z-1} \tag{6.21}$$

The working distance is, of course, the distance covered by the centre of area of the flank perpendicular to the straight side connecting two apexes and any pressure on the rotor flank may be resolved into a force which acts through the centre of area at a variable radius r about the output shaft centre.

$$\int dS = \int r \,.\, d\alpha \quad \text{and} \quad r = e \sin \frac{Z-1}{Z}\alpha$$

the applicable limits for the integral are

$$\alpha_1 = 0 \text{ to } \alpha_2 = \frac{Z \times \pi}{Z-1}$$

$$\text{working distance } S = e\int_{\alpha_1 = 0}^{\alpha_2 = \frac{Z\times\pi}{Z-1}} \sin \frac{Z-1}{Z}\alpha \,.\, d\alpha$$

$$= \left[-e\frac{Z}{Z-1}\cos\left(\frac{Z-1}{Z}\right)\alpha\right]_{\alpha_1 = 0}^{\alpha_2 = \frac{Z\times\pi}{Z-1}}$$

$$= \frac{Z}{Z-1}e\left[1 - \cos\left(\frac{Z-1}{Z}\right)\left(\frac{Z\times\pi}{Z-1}\right)\right]$$

$$= e\frac{2Z}{Z-1}$$

so that

$$V = (A_s + A_{min} - A_r)B_c = R_1 e\, B_c \cdot \frac{4.Z}{Z-1} \sin\frac{\pi}{Z}\ \text{cm}^3 \tag{6.22}$$

This formula simplifies to

$$V = 3\sqrt{3} e R_1 B_c \tag{6.23}$$

for the familiar Wankel configuration, where $Z = 3$ and $\sin\frac{\pi}{3} = \frac{1}{2}\sqrt{3}$.

(The above solution was worked out by Ing. H. Kühner and was first published in MTZ issue 3, March 1964, and reprinted as a leaflet for Technische Entwicklungs Stelle der Fraunhofer Gesellschaft E.V., Lindau.)

Example

The data of the NSU/Wankel KKM 502 engine as fitted to the NSU Spider car are:

$e = 1{\cdot}4$ cm
$R = 10{\cdot}0$ cm
$a = 0{\cdot}2$ cm
$R_1 = 10{\cdot}2$ cm
$\phi = 24^\circ 19'$
$B_c = 6{\cdot}7$ cm

and disregarding the depression in the rotor flanks

$$V = (A_s + A_{min} - A_r)B_c = 3\sqrt{3} e R_1 B_c$$
$$= (A_{max} - A_{min})B_c$$

$$A_{max} = e^2\pi - R_1^2\frac{\sqrt{3}}{4} + R_1^2\frac{\pi}{3} + e \cdot R_1\frac{3\sqrt{3}}{2} \qquad \text{(from Equation 6.11)}$$

$$= 1{\cdot}96\pi - 104{\cdot}04\frac{1{\cdot}732}{4} + \frac{104{\cdot}04\pi}{3} + 1{\cdot}4 \times 10{\cdot}2 \times \frac{3 \times 1{\cdot}732}{2}$$

$$= 107{\cdot}1576\ \text{cm}^2$$

$$A_{min} = e^2\pi - R_1^2 \times \frac{\sqrt{3}}{4} + R_1^2\frac{\pi}{3} - eR_1 \times \frac{3\sqrt{3}}{2} \qquad \text{(from Equation 6.10)}$$

$$= 32{\cdot}95445\ \text{cm}^2$$

$$V = (107{\cdot}1576 - 32{\cdot}95445)6{\cdot}7 = 497{\cdot}16117\ \text{cm}^3$$
$$= 3\sqrt{3} e R_1 B_c \text{ (according to Equation 6.23)}$$
$$= 3 \times 1{\cdot}732 \times 1{\cdot}4 \times 10{\cdot}2 \times 6{\cdot}7 = 497{\cdot}16117\ \text{cm}^3$$