Torsional Resonance Analysis in Air Handling Units

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Many vibration problems and blower failures in small furnaces and air handlers can be eliminated in the design stage by a thorough analysis of incompatibilities in the dynamic properties of the blower assemblies. A theoretical prediction of torsional resonances is developed and design guidelines are presented.

Torsional resonance is one of the vibration problems that causes fatigue failures of wheel-shaft-rotor assemblies. This frequency depends on both the rotor and wheel inertia moments and the shaft stiffness that relies on shaft material properties, shaft size and shaft length between the setscrew and rotor. The size of the shaft flat and tightness of the setscrew are also part of the equation. Vibration problems also involve forces exciting the system. Usually the driving force of the torsional mode is the primary 120 Hz excitation of the motor. But, it's quite possible that 240 Hz excitation may also exist in some applications. A safe margin of the system response to these forces is discussed based on life cycle test results.

Resonances induced by the wheel-shaft-rotor assembly of a blower may result in hub failure, setscrew loosening and shaft gouging. In some extreme cases, the wheel or motor mount may fail. Typical fatigue failures related to setscrew, shaft and hub are discussed. Based on the dynamic properties of the wheel-shaft-rotor assembly, three test methods including both operating and nonoperating vibration tests have been developed to identify the torsional resonance frequency. They are the winding excitation test, strain gage test and impact test. Several techniques can be used to shift the torsional resonance frequency to minimize potential vibration problems and related failures in problematic units.

Wheel-Shaft-Rotor Torsional Frequency

A single-blower system mainly consists of a wheel, a shaft and a motor rotor. This is a generic vibration system. The model is shown in Figure 1. By drawing the free body diagrams and summing the moments on the respective wheel and rotor of the system, the equations of motion for the system are:

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}+\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

where

J_1 = inertia moment of rotor, in. lb sec^2
J_2 = inertia moment of wheel, in. lb sec^2
θ_1 = angular displacement of rotor, rad.
θ_2 = angular displacement of wheel, rad.
k = stiffness of shaft, lb/inch

Solving Equation 1 gives:

$$f=\frac{1}{2\pi}\sqrt{k\left(\frac{1}{J_1}+\frac{1}{J_2}\right)} \quad (2)$$

$$k=\frac{\pi d^4 G}{32L} \quad (3)$$

where

f = torsional natural frequency (Hz)
d = shaft diameter (in.)
G = shear modulus (lb/in^2)
L = shaft length between setscrew and rotor (in.)

In the following case there is a flat on the shaft as shown in Figure 2. The shaft stiffness consists of two parts (k_1 and k_2). As shown in Equation 4, k_1 is the stiffness of the shaft from the section without a flat, while k_2 is the shaft stiffness from the section with a flat. Both of them are given by Equations 5 and 6 respectively.

$$k=\frac{k_1 k_2}{k_1+k_2} \quad (4)$$

$$k_1=\frac{\pi r^4 G}{2(L-l)} \quad (5)$$

$$k_2=\frac{JG}{l} \quad (6)$$

$$J=\frac{\pi r^4}{2}-\frac{r^4}{2}\arccos\left(\frac{r-h}{r}\right)+\frac{r-h}{3}\sqrt{2hr-h^2}\left(\frac{3r^2}{2}+h^2-2hr\right) \quad (7)$$

where

r = Shaft radius (in.)
G = Shear modulus (lb/in^2)

In Equation 7, J is the inertia moment of the flat section. The relationship between the flat height h and flat width w is given by Equation 8.

$$h=\frac{2r-\sqrt{4r^2-w^2}}{2} \quad (8)$$

For a typical steel shaft with 0.5 in. diameter and 4 in. length, the percentage decrease of the shaft stiffness k versus the percentage increase of the flat height h is given in Figure 3 provided l is equal to L. Usually the flat width w is 0.25 in. and its corresponding height h is 0.0335 in., which is 13.4% of the shaft radius. This will cause a 5.12% decrease in the stiffness k and 2.6% decrease in the wheel-shaft-rotor torsional natural frequency.

With a flat width of 0.25 in., the shaft stiffness is a function of the flat length l. The shaft stiffness k versus the percentage increase in the flat length l is shown in Figure 4. This indicates that if the flat length l equals half the shaft length L, the stiffness k will be reduced 2.67%. This causes the torsional natural frequency to decrease by 1.34%.

Setscrew Tightness and Torsional Frequency

A wheel-shaft-rotor assembly, which has a torsional frequency of 118 Hz, was tested with different setscrew tightness values. The setscrew was torqued to 130 in-lb and tested ten times. Each time the wheel-shaft-rotor was reassembled and tightened with the same torque. The final result was averaged by dropping both the highest and lowest frequencies. Then the above tests were repeated eight more times. The torque was reduced 13 in-lb each time. The test results are listed in Table 1 and plotted in Figure 5. The torsional frequency may be reduced by 10% if the setscrew is not tight.

Wheel-Shaft-Rotor Torsional Resonance

Wheel-shaft-rotor torsional resonance usually causes setscrew loosening, shaft gouging and noise problems. In some extremes this mode may also cause other failures. The excitation force of this mode is usually primary torque pulsations at 120 Hz. It could be 240 Hz in some applications. This excitation force and the inertial moment forces from wheel and rotor will cause the setscrew to move in one direction and the shaft in the opposite direction. Both of them shift directions at the same frequency of the torque pulsations. This change causes the setscrew to impact the shaft and induces a reaction pulsation force acting on the hub thread. As the process continues, the impact and fatigue damage of the thread results in setscrew loosening and shaft gouging. At the same time, exces-

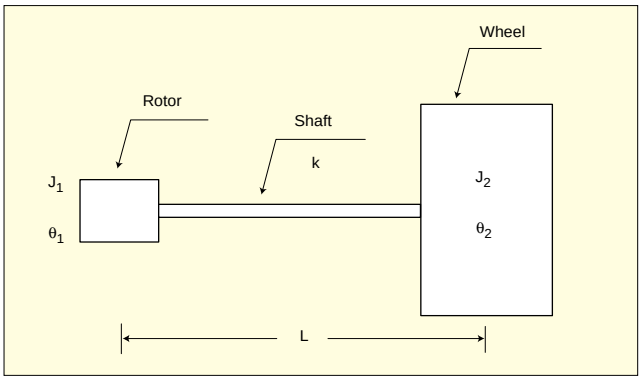


Figure 1. A wheel-shaft-rotor system.

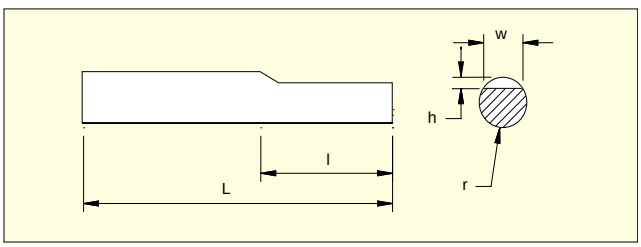


Figure 2. A shaft with a flat.

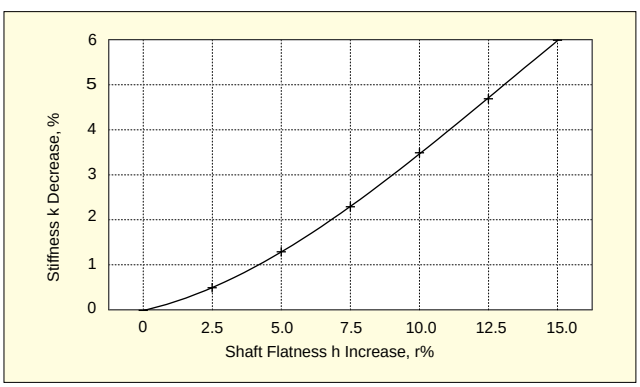


Figure 3. The percentage change of k vs. h.

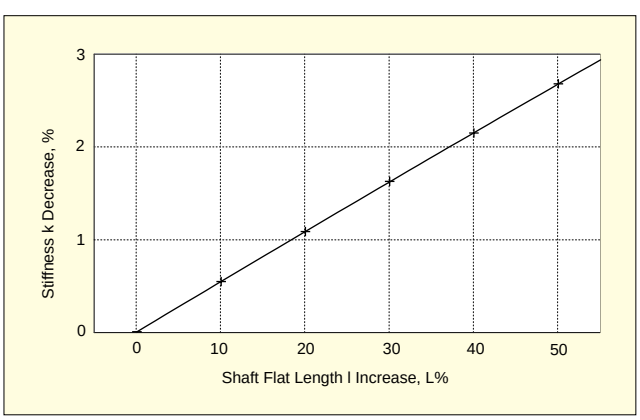


Figure 4. The percentage change of k vs l.

sive noise may be present.

Another type of failure related to the torsional resonance mode is that it induces an excessive force to break the hub loose from the center plate and wear away the hub rollover. Then the center plate moves around on the rotating shaft and cuts the shaft. Figure 6 shows a gouged shaft, a damaged setscrew and wheel. The tip of the damaged setscrew was worn away and some fretting corrosion is also visible. The shaft was cut by the center plate.

The primary torque pulsation excitation in a single-phase PSC motor is 120 Hz. Theoretically speaking, the 4th harmonic order of the power line frequency (240 Hz) will not occur un-

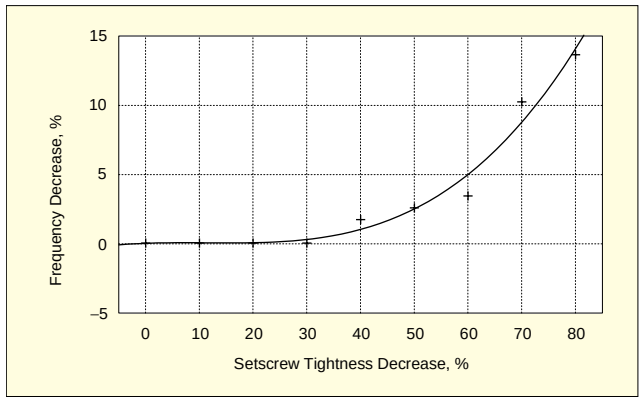


Figure 5. Torsional frequency decrease vs. torque change.

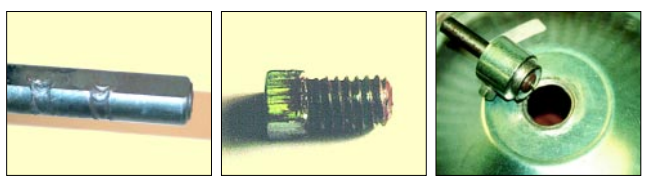


Figure 6. A gouged shaft, a damaged setscrew and damaged wheel.

less the supply power waveforms (either voltage or current) are distorted or truncated. When a speed controller is used to adjust the operating speed it may be possible that the power waveforms are distorted or truncated. According to Fourier series theory, this will generate a 4th harmonic order line frequency torque pulsation excitation at 240 Hz. Usually the amplitude of this excitation is small and will not do any harm to the blower assembly. If the wheel-shaft-rotor system has a torsional response frequency near the excitation frequency, this torque pulsation excitation will be amplified significantly and make excessive noise. An oscilloscope can be used to check if the power waveforms are distorted or truncated.

Test Methods

There are three methods that can be used to test wheel-shaft-rotor torsional natural frequencies. Each of them has its advantages and disadvantages. Different applications may require different test methods.

The winding excitation test is a nonoperating test. This test requires a variable frequency/voltage power source to drive the motor using a pair of diodes. This will prevent rotation but will provide torsional excitation. For a typical single-phase PSC motor, connect one of the motor capacitor wires to one of the diodes while the other motor capacitor wire is idle. No capacitor is required. For motors above 3/4 horsepower, 10 to 20 V will provide the necessary excitation. Increase the excitation voltage accordingly for lower horsepower motors. The bottom line is: *Use enough voltage to excite the system without overheating the test motor.* A winding excitation test is a relatively quick and easy test. There is no need to take the test blower apart. Therefore, the original system assembly conditions are well preserved and the result is usually clear and repeatable. The frequency sweep range is unlimited. In addition to the wheel-shaft-rotor torsional frequency, the motor mount torsional frequency can also be tested at the same time.

The rotor impact test is another nonoperating test method to find the wheel-shaft-rotor torsional natural frequency. This method does not require a signal generator and power amplifier but it does require that both the test blower and the test motor be taken apart.

A strain gage test is required if the rotor and shaft or the wheel hub and shaft have a resilient connection. A resilient

Table 1. Torsional frequency versus setscrew tightness.

Torque, in-lb	130	117	104	091	078	065	052	039	026
Frequency, Hz	118	118	118	118	116	115	114	106	102

connection may create a nonlinear torsional system. The resonance is not only a function of the system parameters but is also related to the excitation forces. Therefore, this resonance must be measured at the operating condition. In this method, the motor torque pulsation serves as an excitation force. For a 60 Hz motor, the power line frequency can go as high as 75 Hz without overheating the motor. If the resonance is close to 240 Hz, a truncated sine wave power source may be used to induce 4th harmonic order of the line frequency torque pulsation. This test can be conducted with a variable frequency/voltage power source and by mounting two torque strain gages on the shaft between the setscrew and rotor parallel to the shaft. The strain gage test requires experience and skill in order to get satisfactory results.

A wheel-shaft-rotor system was tested using the above three methods. The results are plotted in Figures 7, 8, and 9.

Safety Ranges of the Response Frequency

Most direct drive blowers use a single-phase PSC motor with 120 Hz torque pulsation. In some applications, side band excitations at 120 Hz ±RPM/60 may also cause torsional vibration and noise problems if the system response frequency is close to one of these excitation frequencies. Based on experience and life cycle test results, the torsional natural frequency f_t of a wheel-shaft-rotor system will be okay if Inequality 9 is satisfied.

$$x_{120}\times\left(120-\frac{\text{RPM}}{60}\right)>f_t>1.14\times\left(120+\frac{\text{RPM}}{60}\right) \qquad (9)$$

Where

$$x_{120}=\frac{240-1.14\times\left(120+\frac{\text{RPM}}{60}\right)}{120-\frac{\text{RPM}}{60}} \qquad (10)$$

x_{120} is calculated based on the same percentage increase and decrease at both side bands.

In the case when a 240 Hz torque pulsation also exists, both Inequalities 9 and 11 should be satisfied.

$$x_{240}\times\left(240-\frac{\text{RPM}}{60}\right)>f_t>1.1\times\left(240+\frac{\text{RPM}}{60}\right) \qquad (11)$$

Where

$$x_{240}=\frac{480-1.1\times\left(240+\frac{\text{RPM}}{60}\right)}{240-\frac{\text{RPM}}{60}} \qquad (12)$$

The values of x_{120}, x_{240}, and f_t limits at 120 and 240 Hz are listed in Table 2.

For a six-pole motor, as an example, the torsional natural frequency f_t of the wheel-shaft-rotor system should satisfy both Inequalities 13 and 14.

$$f_t \neq 80 \text{ to } 160 \text{ (Hz)} \qquad (13)$$

$$f_t \neq 194 \text{ to } 286 \text{ (Hz)} \qquad (14)$$

The safety factors in Inequalities 9 (14%) and 11 (10%) are chosen to avoid side band excitation forces that may be a problem in some applications. For those applications without side band concern these safety factors may be a little conservative.

Solutions

Resonance problems involve the forces exciting the system and the response of the system to these forces. Different systems respond to the same exciting force differently. If a force acts on the wheel in a blower assembly, the vibration of the wheel may be present. If the same force is applied to a very solid concrete wall, nothing will happen. A system responds to a cyclic excitation force like a force amplifier. When the frequency of the excitation force coincides with the system natural frequency, this force will be greatly amplified. The factor of the force amplified by the system is called response function. Thus one could, in principle, get rid of any objectionable

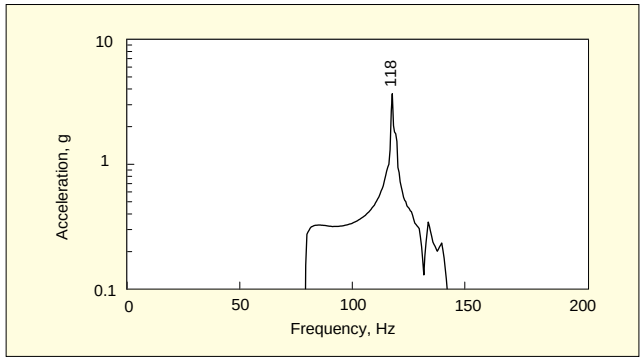


Figure 7. Winding excitation test result.

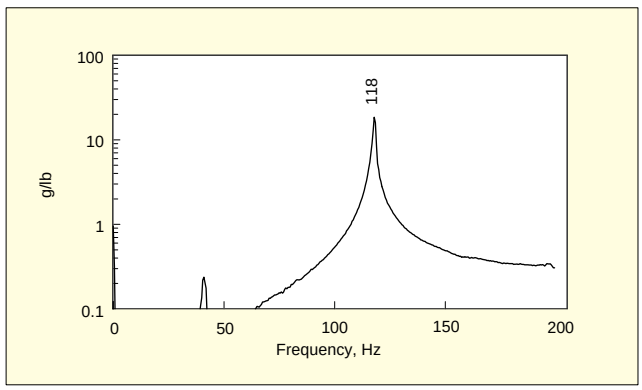


Figure 8. Rotor impact test result.

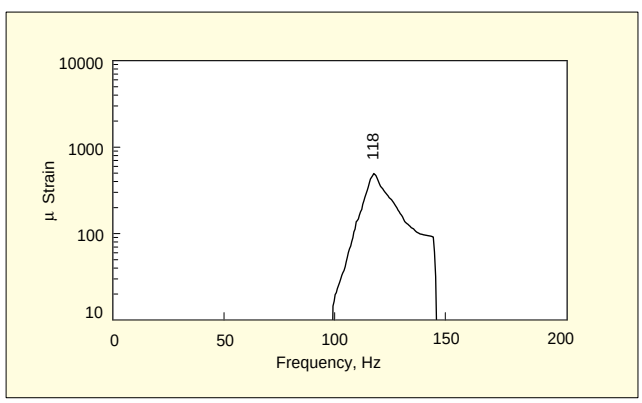


Figure 9. Strain gage test result.

vibration, stress or noise either by reducing the exciting force or by reducing the response function of the system. Experience tells us that a reduction of the exciting force tends to be much more costly than a reduction of the system response function. Therefore the following discussions are focused on the wheel-shaft-rotor torsional response.

The wheel-shaft-rotor torsional natural frequency depends on the wheel, rotor and shaft length between the setscrew and rotor. It is necessary to find out how these components affect

Table 2. x values and f_t limits.

Motor Poles	**4**	**6**	**8**	**10**	**12**
RPM	1800	1200	900	720	600
x_{120}	0.767	0.804	0.82	0.829	0.834
x_{240}	0.871	0.882	0.887	0.89	0.892
f_{b120}	69	80	86	90	92
f_{t120}	171	160	154	151	148
f_{b240}	183	194	200	203	205
f_{t240}	297	286	281	277	275

f_{b120} = bottom limit of f_t at 120 Hz ($f_{b120} > f_t$)
f_{t120} = top limit of f_t at 120 Hz ($f_t > f_{t120}$)
f_{b240} = bottom limit of f_t at 240 Hz ($f_{b240} > f_t$)
f_{t240} = top limit of f_t at 240 Hz ($f_t > f_{t240}$)

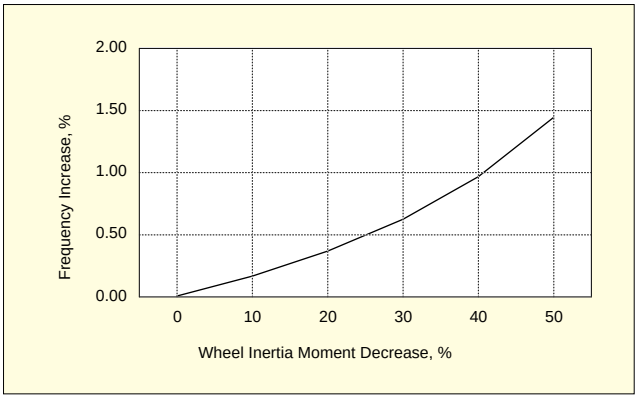


Figure 10. Frequency increase vs. wheel inertia moment decrease.

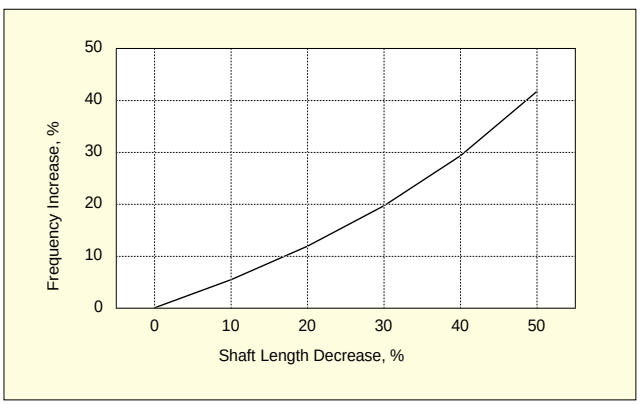


Figure 11. Frequency increase vs. shaft length decrease.

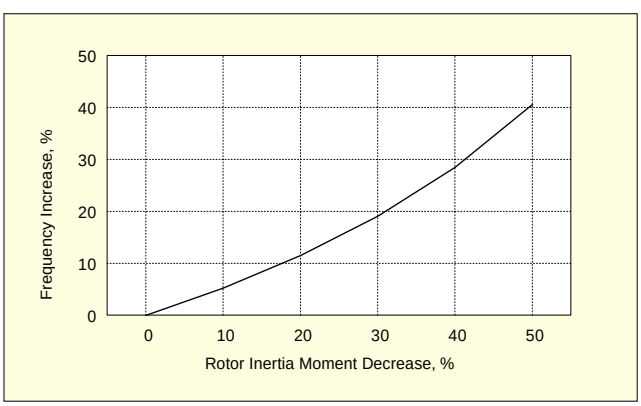


Figure 12. Frequency increase vs. rotor inertia moment decrease.

the system response. Then different solutions may be available for different applications. Let's look at a typical wheel-shaft-rotor assembly with the following parameters: wheel diameter = 11 in; wheel width = 10 in; wheel inertia = 0.472 in-lb sec^2; rotor inertia = 0.0142 in-lb sec^2; shaft diameter = 0.5 in; shaft length = 4 in; shaft shear modulus = 115 × 105 lb/in^2.

The torsional natural frequency of this system is 180 Hz calculated by Equation 1. If the wheel inertia moment J_2 in Equation 1 is reduced, the torsional frequency will increase. Figure 10 shows that a 50% decrease in the wheel moment of inertia provides only a 1.4% increase in the torsional natural frequency which indicates the wheel does not affect the system response very much. This is because the wheel inertia moment is much higher than that of the rotor.

When the shaft length between the setscrew and rotor is reduced, this torsional natural frequency will dramatically increase as shown in Figure 11. Replacing a convex wheel with a concave wheel or moving the motor closer to the setscrew will shorten the shaft length. This is a very handy technique to solve torsional resonance problems. If the shaft length decreases 50%, the frequency increases 42%.

Figure 12 shows the comparison of frequency versus the rotor

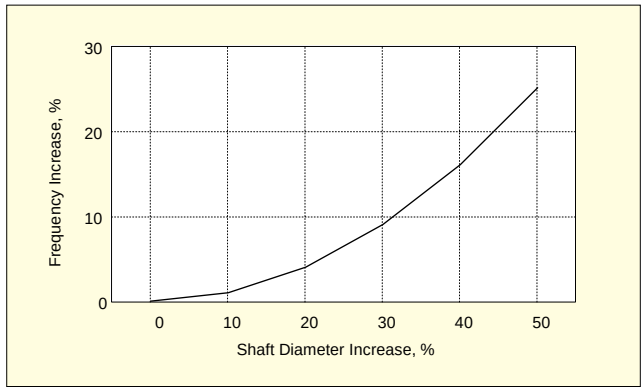


Figure 13. Frequency increase vs. shaft diameter increase.

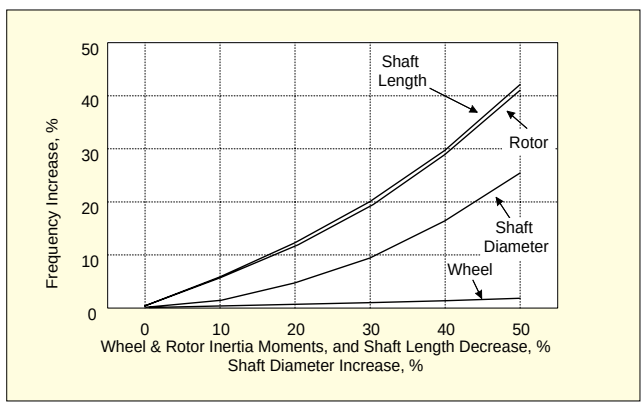


Figure 14. Frequency increase vs. blower component change.

inertia moment. If the rotor inertia decreases 50%, the frequency increases 41%. This means that this torsoinal natural frequency is also very sensitive to a change in the rotor inertia moment. This is why sometimes one motor works okay while another one may not.

Increasing the shaft diameter will also affect the torsional natural frequency. If the shaft diameter is increased 50%, the frequency will increase 25% which is plotted in Figure 13.

Each system has its own characteristic curve (see Figure 14). In this assembly, the shaft length between the setscrew and rotor affects the torsional resonance most while changes in the wheel contribute very little to the response. In other words the torsional resonance problems result from the system dynamics instead of from the wheel itself.

Conclusions

Wheel-shaft-rotor torsional natural frequency mainly depends on the shaft length between the setscrew and rotor, rotor inertia moment and shaft diameter. The changes in wheel inertia moment don't affect this frequency very much. The reason is because the wheel inertia moment is much higher than that of the rotor.

Changing the shaft length between the setscrew and rotor is one of the most effective ways to shift the wheel-shaft-rotor torsional frequency. Switching to an inverted centerplate wheel or moving the motor closer to or away from the setscrew will change this shaft length. It is possible that using a different type of motor may cause the wheel-shaft-rotor torsional frequency to dramatically shift even though the shaft diameter, shaft length and wheel are the same. For a typical 0.5 in. diameter shaft with 0.25 in. flat width, the shaft stiffness k reduces 5.12% and the wheel-shaft-rotor torsional frequency decreases 2.6%. Setscrew tightness affects the wheel-shaft-rotor torsional frequency. It could cause this frequency to decrease as much as 10%.

The winding excitation test is relatively quick and easy. The torsional frequency of the motor mounting system can also be tested at the same time. For a nonlinear system a strain gage test is the best choice.