BY PAUL LAMAR

More about CORNERING POWER

What you can learn on a skid pad about the handling of your car

THE ARTICLE ON ultimate cornering power by Ron Wakefield in the April 1969 issue of *Road & Track* has encouraged me to set down some of the basic physical laws and equations that govern the maximum cornering forces a car can develop in a steady-state turn. As that article pointed out, this is not the whole story to handling, but nevertheless it remains a large part.

To begin at the beginning—that is, the tire-road contact —passenger-car tires develop their maximum cornering force at approximately 12° slip angle on a given road surface; this angle does vary, however, with type of tire, load, inflation pressure, rim width, rubber compound, camber angle and torque applied to the tire. Fig. 1 defines several of the terms I will be using in this discussion.

A driver in the process of cornering cannot tell what his precise front and rear slip angles are, but if he is sensitive he'll know when he has reached maximum cornering force. Since the mathematical equations defining cornering power are considerably simpler at the optimum slip angle, I will assume for the purposes of this article that the slip angles are optimum for maximum cornering power under the given conditions. I will also assume that the maximum lateral coefficient of friction for a non-driven tire (a front tire on a rear-drive car at optimum slip and camber angle) is inversely related to the vertical load on that tire as shown in Fig. 2. The equation for this line in Fig. 2 is

$$f = b - mN \qquad (1)$$

where b is the maximum value of the lateral coefficient of friction, occurring when the vertical load N equals zero. For present tires, b varies from 1.1 to 1.6. The value of m is a measure of how sensitive this lateral coefficient of friction of the tire is to tire load; m varies between 0.0002 and 0.0006 for passenger-car tires. This contradicts the common misconceptions that (a) the coefficient of friction cannot exceed one and (b) does not depend on load. As the activities of the drag strip boys prove every weekend, the coefficient of friction of a rubber tire on pavement can substantially exceed 1.

The cornering force a tire will generate at optimum slip and camber angle is proportional to the load on the tire and that tire's lateral coefficient of friction at that given load. The equation for cornering force for one tire is therefore $CF = fN$ which is the same as $CF = (b\text{-}mN)N$. If we combine the load-sensitive characteristics of the inside and outside front tires of a car negotiating a skid pad and plot the total cornering force of both tires as it is related to the load transfer from inside to outside tire, we get a curve such as Fig. 3. The equation for Fig. 3 is

$$CF = (b - mN_i)N_i + (b - mN_o)N_o \qquad (2)$$

N_i and N_o are inside and outside loads respectively. Fig. 3 shows that as you increase load transfer, the total cornering force at the front of the car is reduced and this causes the car to understeer more.

The situation for the rear (drive) tires is much more complicated. In addition to being load sensitive, the drive tires' lateral coefficient of friction is sensitive to rear axle torque or tire patch thrust as well, decreasing with driving or braking thrust. This is what causes the well known power oversteer effect. In order to relate this friction coefficient sensitivity to thrust mathematically we must make further generalizations; therefore, another simplifying assumption I will make is that the maximum coefficient of friction in any direction (lateral, fore or aft) relative to the tire contact patch at the optimum slip and camber angle is the same. For example, if you are using the maximum lateral coefficient to corner with, then there's nothing left to accelerate with and vice versa. That is to say, if you tried to apply too much torque or accelerative thrust while you were cornering, you ➡

Fig. 1. Tire forces, directions and angles. Slip angles are exaggerated for clarity. Chassis is shown in an extreme understeering attitude with driving thrust applied.

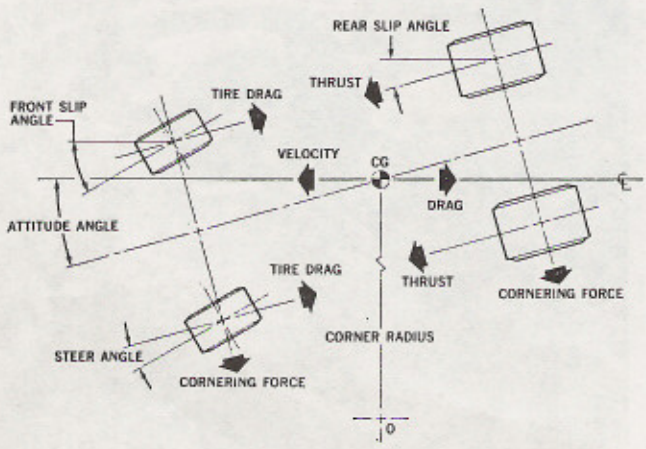
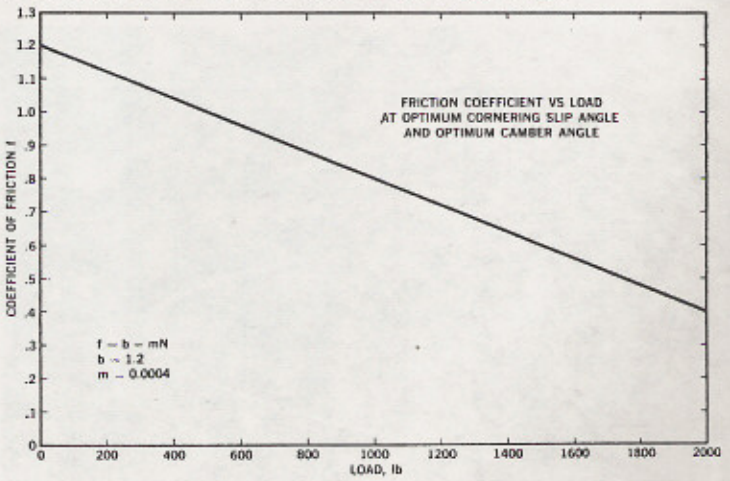


Fig. 2. The coefficient of friction of a tire on pavement is not constant but rather decreases with increasing load. This is why tire size must increase as weight does.



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would merely spin; conversely, if your rear tires were delivering their maximum accelerative thrust and you jerked the steering wheel, you'd also spin.

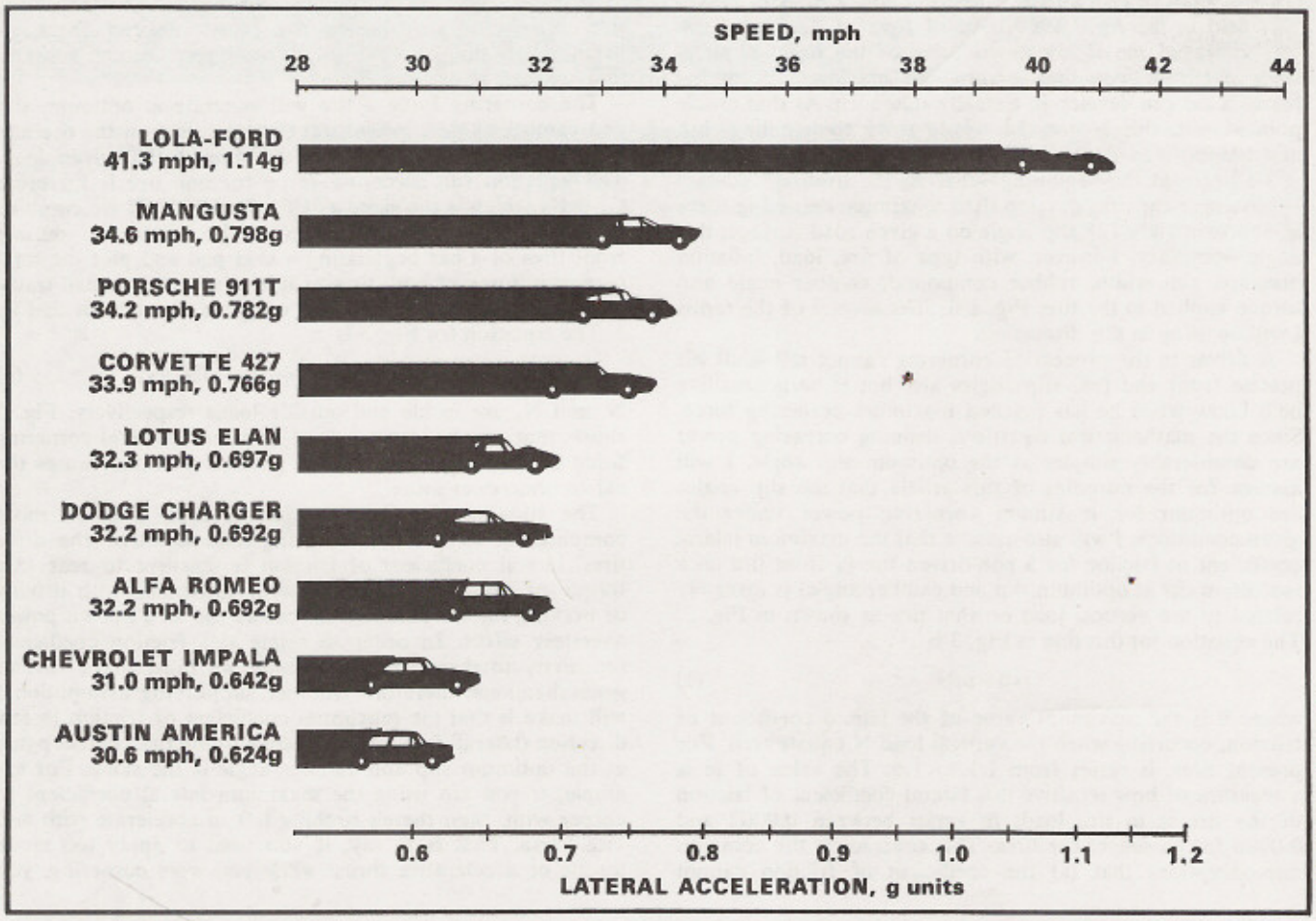
Stating this mathematically, the equation for the cornering force of one rear drive wheel as it is related to load and thrust is

$$CF = \sqrt{(fN)^2 - T^2}$$

which is the same as

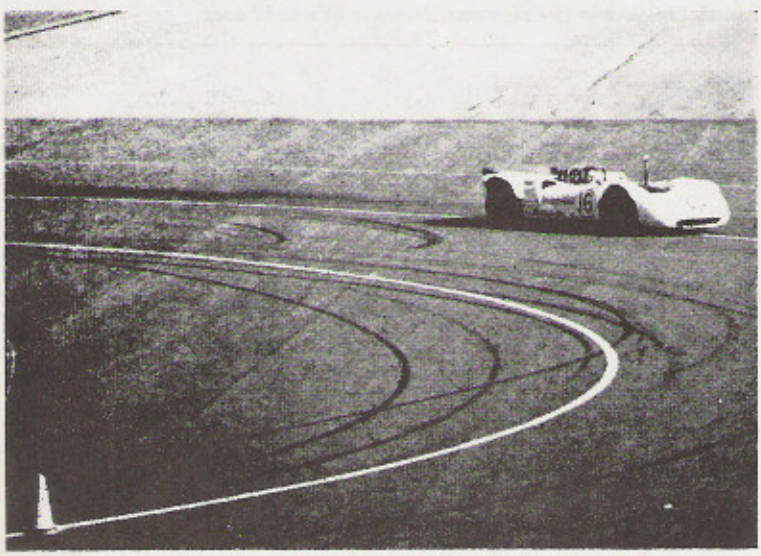
$$CF = \sqrt{[(b - mN)N]^2 - T^2} \qquad (3)$$

where T represents the contact patch thrust in pounds.



ILLUSTRATIONS BY BOB PRICE

Cornering performance of the nine cars tested for the April story on cornering power. Below, the Lola-Ford and the Austin.





Combining the equation for inside and outside drive wheels gives an equation like this:

$$CF = \sqrt{[(b-mN_i)N_i]^2-T^2} + \sqrt{[(b-mN_o)N_o]^2-T^2} \quad (4)$$

The thrust T is the same for both drive wheels with a conventional differential because conventional differentials divide the torque equally between the drive axles. Limited-slip differentials modify this equation considerably when inside drive-wheel spin is a severe problem. Fortunately, most properly set-up cars will not suffer from this problem and we can ignore this effect.

To appreciate the meaning of equation (4) in terms of actual vehicle cornering performance, we want to "plug into it" different values for each variable such as weight transfer, thrust and total weight on the driving wheels. This is a cumbersome job for slide rule, pencil and paper, so this has been done by a general-purpose digital computer at Business Methods and Software, Inc., Lawndale, Calif. The results of this computer study of Equation (4) are shown in Fig. 4, which relates the total cornering force available (in terms of lateral acceleration in g units) to weight transfer and thrust for the driven end of a car—i.e., the rear of a rear-drive car or the front of a front-drive. The straight lines in Fig. 4 (weight transfer lines) relate weight transfer from inner to outer wheel to the lateral acceleration, for a specific car with several anti-roll bar sizes at the driven end. To imagine how Fig. 4 might be used in determining the cornering potential of a certain car, take the following example:

We want to try a 0.9-in. anti-roll bar at the rear of our Group 7 car, and we are trying to achieve a 1.2-g cornering capability. Reading from the weight transfer line for the 0.9-in. bar, we find that this condition corresponds to a weight transfer of 355 lb and a driving-wheel thrust of approximately 110 lb. Thus we would be limited to only 110 lb thrust at the driving wheels, which isn't very much thrust. In other words, the driving wheels couldn't corner at 1.2 g at any speed where more than 110 lb thrust was required to drive the car. Obviously some means of adding downforce to the driving wheels is called for here, and that's where wings come in. This example should also serve to demonstrate why, without the help of aerodynamic devices for downforce, a rear-drive car tends toward oversteer as speed goes up.

The point at which a given weight transfer line crosses a curve is the cornering power as it relates to thrust and this

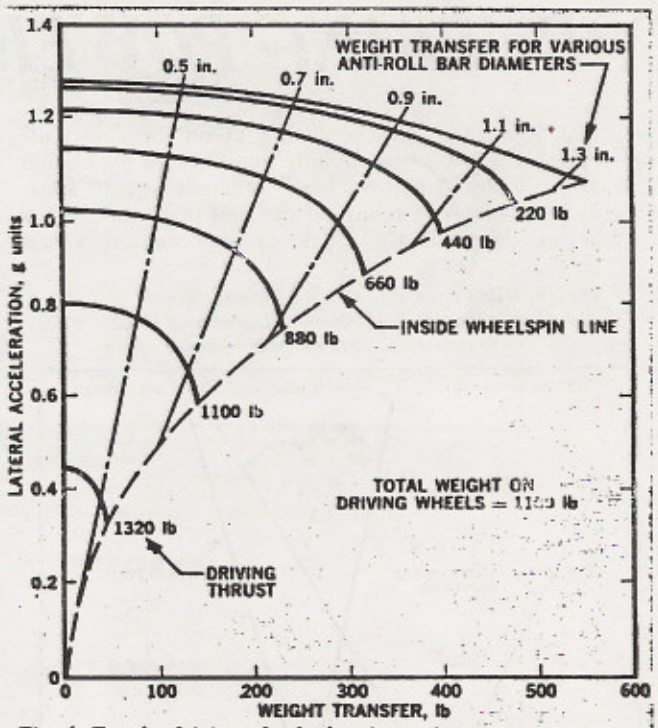


Fig. 4. For the driving wheels the picture is more complex. Here the lateral acceleration available is plotted against driving thrust and weight (or load) transfer.

is plotted in Fig. 5, which is merely a particular case of Fig. 4. Inside drive-wheel spin occurs with a conventional differential at the termination point of the curves, so that no further increase in cornering speed can result. It can be seen by comparing Fig. 3 with Fig. 5 that load transfer has a much greater detrimental effect on the driving end than on the non-driving end and that the driving wheels usually put the limiting factor on maximum cornering power. The importance of thrust in these equations is twofold, then: a considerable amount is required to scrub the non-driving tires around the skid pad, and the delivery of thrust takes away from the cornering capability of driving tires.

So much for theory. Why won't the Austin America ➡

Fig. 3. Weight transfer robs a pair of tires of cornering power. This is a plot of total cornering force available from the two non-driving wheels as weight transfer increases.

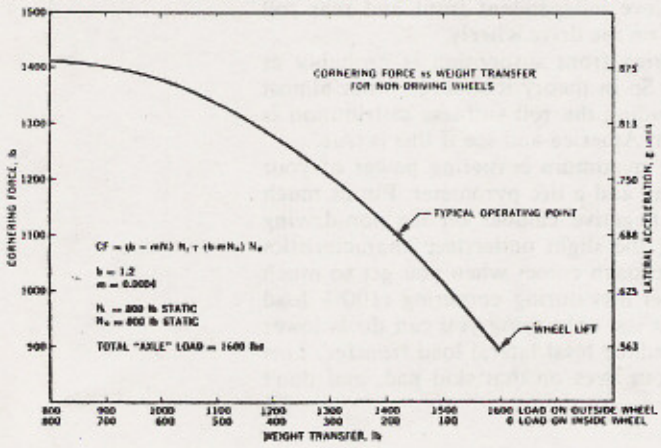
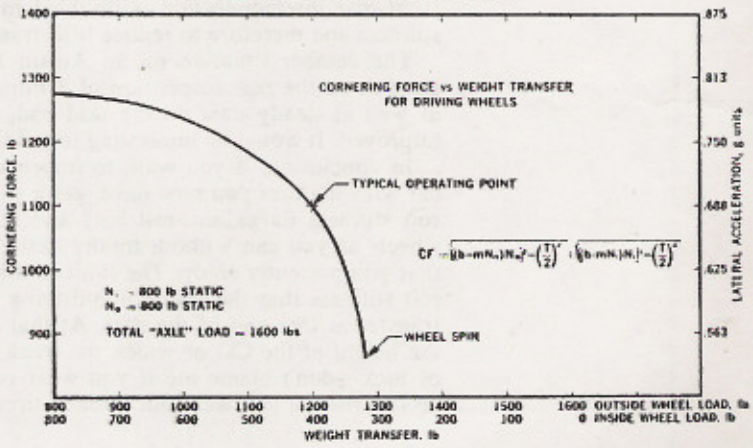


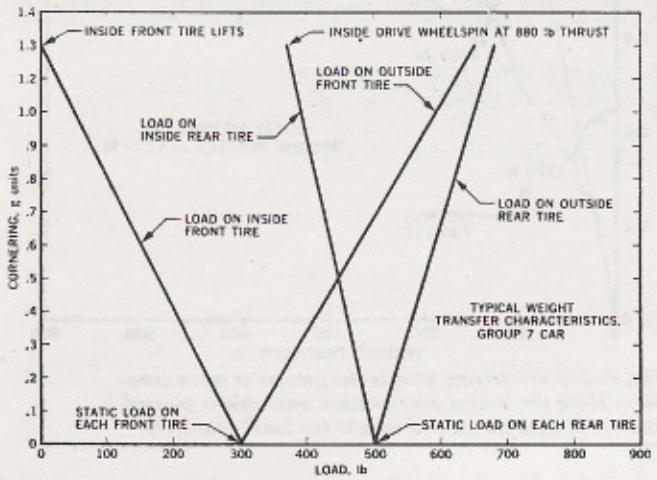
Fig. 5. For a chosen anti-roll bar diameter Fig. 4 can be converted into this simplified curve. Here cornering force is used instead of lateral acceleration values.



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(0.624 g) stay with a Porsche (0.782 g) on the skid pad? Well, it turns out (by an incredibly complicated evaluation of the above equation) that for many different fore-aft static weight distributions (i.e., relative front and rear anti-roll bar sizes) the hot setup for a given set of tires and car weight is: most of the car's static weight on the drive wheels (be they front *or* rear), most of the roll stiffness on the non-driven wheels, and near-neutral steer. Coincidentally, this situation is precisely what one obtains in most modern racing cars and has been empirically developed over a period of many years. Fig. 6 is a graph showing the weight transfer and weight distribution characteristics of a typical rear-engine race car. The reason for having a larger percentage of weight, other than for reasons of maximum forward acceleration, on the drive wheels is that taking it off the non-driving wheels lowers the force required to scrub them around the skid pad—a force which must come from the thrust at the drive wheels' contact patches. Furthermore, a given thrust affects a heavily loaded tire less than a lightly loaded tire because it is a smaller percentage of the total thrust the tire can transmit.

Fig. 6. Weight transfer as a function of lateral acceleration for a car with 600 lb on the front tires and 1000 lb on the rear; note that total load on pair remains same.



A further effect that clouds the issue is the non-optimum camber characteristics of most cars. The camber angle, and therefore contact patch efficiency, can be easily checked on a specific car after several laps around the skid pad by means of a tire pyrometer (a tire pyrometer is just a quick acting thermometer). The difference in the temperature between inside and outside of the tire tread speaks volumes about the camber at maximum cornering speed; ideally we want the same temperature all the way across the tire tread. Tire temperature can also indicate an under- or over-inflated tire.

But back to that question about the Austin America. According to the R&T article, the tire loading of an Austin America is better than a De Tomaso Mangusta which corners at 0.798 g, and according to our theory, it has a desirable percentage of its total weight on the driving wheels. It must be either the roll stiffness distribution or the camber (suspension geometry characteristics). Let's look at the roll stiffness distribution first.

The Austin America's Hydrolastic suspension gives a good ride and a fast swerve response but does not allow the needed appreciable difference in the front and rear roll stiffness for maximum cornering speeds; its front-to-rear hydraulic interconnection tends to *equalize* the front and rear roll stiffness. This fact has been demonstrated by recent racing history in England: when a Mini, late models of which have similar suspension to that of the Austin America, is raced, its front-rear interconnection is disabled to achieve independent front and rear roll stiffness and therefore to reduce load transfer on the drive wheels.

The camber situation on an Austin America front suspension is probably as good as on the rear suspension of a Porsche. So in theory it should corner almost as well at steady state on the skid pad, providing the roll stiffness distribution is improved. It would be interesting to modify an America and see if this is true.

In conclusion, if you want to improve the maximum cornering power of your car with the tires you now have, get a skid pad and a tire pyrometer. Put as much roll stiffness (large anti-roll bar) and static negative camber on the non-driving wheels as you can without totally destroying the slight understeer characteristics that you presently enjoy. The limit to this approach comes when you get so much roll stiffness that the inside non-driving wheel lifts during cornering (100% load transfer at that end of the car). At that point the only thing you can do is lower the height of the CG or widen the track to reduce total lateral load transfer. Lots of luck—don't blame me if you wear out your tires on that skid pad, and don't be surprised if you wear out a set of tires.